

Atwood Primary School Calculations Policy



*September 2010
Review September 2013*

Introduction

This policy contains the key pencil and paper procedures that will be taught at Atwood Primary School. It has been written to ensure consistency and progression throughout the school.

Although the focus of the policy is on pencil and paper procedure, it is important to recognise that the ability to calculate mentally lies at the heart of the Primary National Strategy for mathematics. The mental methods in the *Primary Framework for teaching mathematics* will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop these skills.

Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of a calculator.

This policy concentrates on the introduction of standard symbols, the use of empty number lines as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation or problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task. They will do this by always asking themselves:

‘Can I do this in my head?’

‘Can I do this in my head using drawings or jottings?’

‘Do I need to use a pencil and paper procedure?’

‘Do I need a calculator?’

Objectives

Foundation Stage Early learning goals for 'Calculating'

- * Share objects into equal groups and count how many in each group.
- * In practical activities and discussion, begin to use the vocabulary involved in adding and subtracting.
- * Use language such as 'more' or 'less' to compare two numbers.
- * Find one 'more' or one 'less' to compare two numbers.
- * Begin to relate addition to combining two groups of objects and subtraction to 'taking away'.

Year 1 to Year 6

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

Using and applying mathematics	Calculating
<p>Year 1</p> <ul style="list-style-type: none"> • Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change' • Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context 	<p>Year 1</p> <ul style="list-style-type: none"> • Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one digit or two-digit number • Understand subtraction as 'take away and find a <i>'difference'</i> by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number • Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
<p>Year 2</p> <ul style="list-style-type: none"> • Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence • Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem 	<p>Year 2</p> <ul style="list-style-type: none"> • Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders • Use the symbols +, -, x, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. $D + 2 = 6$, $30 - D = 24$)
<p>Year 3</p> <ul style="list-style-type: none"> • Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations • Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure 	<p>Year 3</p> <ul style="list-style-type: none"> • Develop and use written methods to record, support or explain addition and subtraction of two digit and three-digit numbers. • Use practical and informal written methods to multiply and divide two digit numbers (e.g. 13×3); round remainders up or down, depending on the context. • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences

Using and applying mathematics	Calculating
<p>Year 4</p> <ul style="list-style-type: none"> • Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate • Represent a <i>puzzle</i> or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem 	<p>Year 4</p> <ul style="list-style-type: none"> • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Develop and use written methods to <i>record</i>, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15×9, $98 \div 6$)
<p>Year 5</p> <ul style="list-style-type: none"> • Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use • Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem 	<p>Year 5</p> <ul style="list-style-type: none"> • Use efficient written methods to add and subtract whole numbers and decimals with up to two places. • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 • Refine and use efficient written methods to multiply and divide HTU \times U, TU \times TU, U.t \times U and HTU \div U
<p>Year 6</p> <ul style="list-style-type: none"> • Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use • Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of n pens at 15 pence each is $15n$ pence) 	<p>Year 6</p> <ul style="list-style-type: none"> • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

Mental methods of calculation

Up to Year 3, the emphasis should be on children working mentally. Once written methods are introduced, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of number bonds

$$6 + 4 = 10$$

$$\square + 3 = 10$$

$$25 + 75 = 100$$

$$19 + \square = 20$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 6 times table
 10 times table

Year 4 Derive and recall all multiplication facts up to 10×10

Years 5 & 6 Derive and recall quickly all multiplication facts up to 10×10 .

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$13 \times 11 = (13 \times 10) + (13 \times 1)$
 $= 130 + 13$
 $= 143$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$23 \times 4 = (20 \times 4) + (3 \times 4)$
 $= 80 + 12$
 $= 102$

Use of factors

$8 \times 12 = 8 \times 4 \times 3$

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 6 times table
 10 times table

Year 4 Derive and recall division facts for all tables up to 10 x 10

Year 5 & 6 Derive and recall quickly division facts for all tables up to 10 x 10

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$378 \div 21$ $378 \div 3 = 126$ $378 \div 21 = 18$
 $126 \div 7 = 18$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div$

Moving On

How can children's readiness for written calculations be judged?

Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:

- Do children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply the commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children know the 2, 3, 4, 5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand place value?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Reception

Addition

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.
They develop ways of recording calculations using pictures, etc.



Bead strings or bead bars can be used to illustrate addition



$8 + 2 = 10$

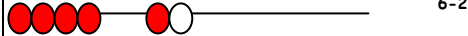
They use numberlines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.

Subtraction

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.
They develop ways of recording calculations using pictures etc.



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.



$6 - 2 = 4$

They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

Multiplication

Children will experience equal groups of objects.

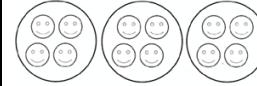
They will count in 2s and 10s and begin to count in 5s.

They will work on practical problem solving activities involving equal sets or groups.



Division

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.



ADDITION

YEAR 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$\begin{aligned} 2 &= 1 + 1 \\ 2 + 3 &= 4 + 1 \\ 3 &= 3 \\ 2 + 2 + 2 &= 4 + 2 \end{aligned}$$

Missing numbers need to be placed in all possible places.

$$\begin{array}{ll} 3 + 4 = \square & \square = 3 + 4 \\ 3 + \square = 7 & 7 = \square + 4 \\ \square + 4 = 7 & 7 = 3 + \square \\ \square + \nabla = 7 & 7 = \square + \nabla \end{array}$$

Activities

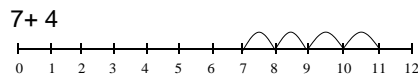
Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.

Teacher modelling

Drawing jumps on numbered number lines to support understanding of the mental method

Children

To create their own jumps using rulers, fingers, pens, bodies etc.



YEAR 2

+ = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

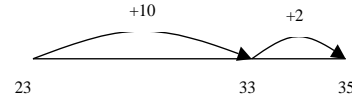
Extend to
 $14 + 5 = 10 + \square$
 and
 $32 + \square + \square = 100 \quad 35 = 1 + \square + 5$

Partition into tens and ones and recombine

$$\begin{aligned} 12 + 23 &= 10 + 2 + 20 + 3 \\ &= 30 + 5 \\ &= 35 \end{aligned}$$

Count on in tens and ones

$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g.

Children should be able to partition the 7 to relate adding the 2 and then the 5.

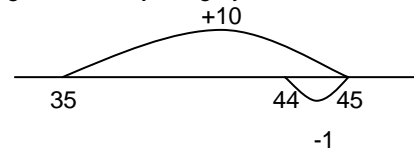
$$8 + 7 = 15$$



Add 9 or 11 by adding 10 and adjusting by 1

e.g.

Add 9 by adding 10 and adjusting by 1
 $35 + 9 = 44$



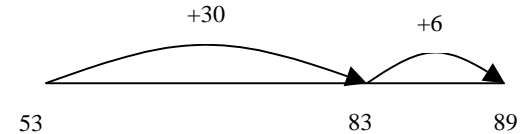
YEAR 3

+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

- Partition both numbers and recombine.
- Count on by partitioning the second number only e.g.
 $36 + 53 = 53 + 30 + 6$
 $= 83 + 6$
 $= 89$

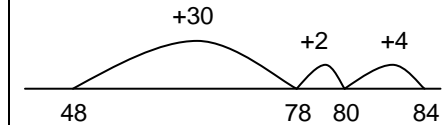


Add a near multiple of 10 to a two-digit number

Secure mental methods by using a number line to model the method. Continue as in Year 2 but with appropriate numbers e.g. $35 + 19$ is the same as $35 + 20 - 1$.

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

$$48 + 36 = 84$$



pencil and paper procedures

$$83 + 42 = 125$$

either

or

1. Vertical expansion

$$\begin{array}{r} 83 \\ + 42 \\ \hline 5 \\ \hline 120 \\ \hline 125 \end{array}$$

2. Horizontal expansion

$$\begin{array}{r} 80 + 3 \\ + 40 + 2 \\ \hline 120 + 5 = 125 \end{array}$$

ADDITION

YEAR 4

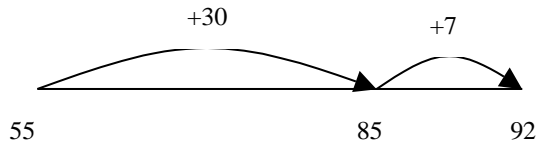
+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Partition into tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$



Add the nearest multiple of 10, then adjust

Continue as in Year 2 and 3 but with appropriate numbers e.g. $63 + 29$ is the same as $63 + 30 - 1$

Pencil and paper procedures (starting with no carrying)

$$367 + 185 = 431$$

either or

$\begin{array}{r} 367 \\ +185 \\ \hline 12 \\ 140 \\ 400 \\ \hline 552 \end{array}$	$\begin{array}{r} 300 + 60 + 7 \\ 100 + 80 + 5 \\ 400 + 140 + 12 = 552 \end{array}$
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leading to carrying by the end of the year

$\begin{array}{r} 367 \\ +185 \\ \hline 552 \\ \small{11} \end{array}$	$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ \small{1} \end{array}$	$\begin{array}{r} 783 \\ \quad 42 \\ \hline 825 \\ \small{1} \end{array}$
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Extend to decimals in the context of money.

$£2.50 + £1.75 =$	$2.00 + 50$	$£2.50$
	$1.00 + 70 + 5$	$£1.75$
	$3.00 + 1.20 + 5 = 4.25$	$£4.25$

YEAR 5

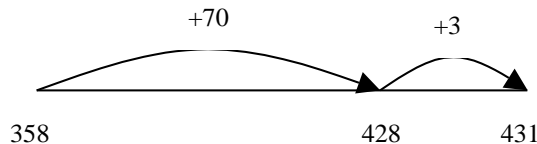
+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Partition into hundreds, tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$



Add or subtract the nearest multiple of 10 or 100, then adjust

Continue as in Year 2, 3 and 4 but with appropriate numbers e.g. $458 + 79 =$ is the same as $458 + 80 - 1$

Pencil and paper procedures

Extend to numbers with at least four digits

$$3587 + 675 = 4262$$

$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \small{111} \end{array}$	$\begin{array}{r} 587 \\ +475 \\ \hline 1062 \\ \small{11} \end{array}$
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Revert to expanded methods if the children experience any difficulty.

Extend to up to two places of decimals (same number of decimal places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ +54.6 \\ \hline 127.4 \\ \small{11} \end{array}$$

YEAR 6

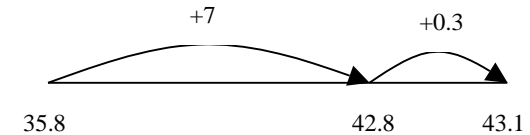
+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 35.8 + 7.3 &= 35.8 + 7 + 0.3 \\ &= 42.8 + 0.3 \\ &= 43.1 \end{aligned}$$



Add the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Year 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc

Pencil and paper procedures

Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.

$$13.86 + 9.481 = 23.341$$

$\begin{array}{r} 13.86 \\ + 9.481 \\ \hline 23.341 \\ \small{111} \end{array}$	$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ \hline +4681 \\ 11944 \\ \small{121} \end{array}$
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Revert to expanded methods if the children experience any difficulty.

Subtraction - continued

YEAR 1

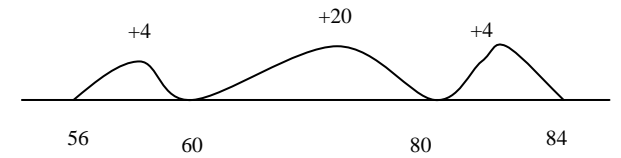
YEAR 2

YEAR 3

Pencil and paper procedures

Complementary addition

$84 - 56 = 28$



$74 - 27$

$326 - 178$

$$\begin{array}{r} 74 \\ - 27 \\ \hline +3 = 30 \\ +40 = 70 \\ \hline +4 = 74 \\ 47 \end{array}$$

$$\begin{array}{r} 326 \\ - 178 \\ \hline +2 = 180 \\ +20 = 200 \\ \hline +126 = 326 \\ 148 \end{array}$$

Subtraction

YEAR 4

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a small difference by counting up

e.g. $5003 - 4996 = 7$

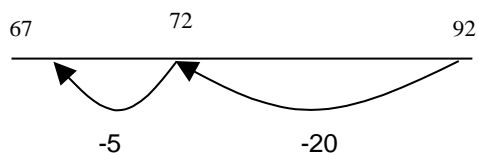
This can be modelled on an empty number line (see complementary addition below). Children should be encouraged to use known number facts to reduce the number of steps.

Subtract the nearest multiple of 10, then adjust.

Continue as in Year 2 and 3 but with appropriate numbers.

Use known number facts and place value to subtract

$92 - 25 = 67$



$754 - 86 = 668$

$700 + 50 + 4$

80 + 6

$700 + 40 + 14$

80 + 6

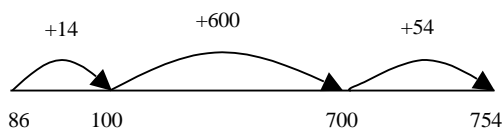
$600 + 140 + 14$

80 + 6

$600 + 60 + 8 = 668$

Pencil and paper procedures

Complementary addition



YEAR 5

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $8006 - 2993 = 5013$

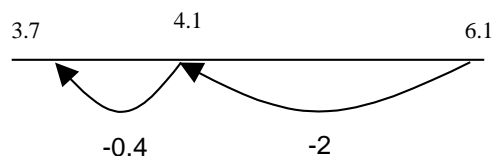
This can be modelled on an empty number line (see complementary addition below).

Subtract the nearest multiple of 10 or 100, then adjust.

Continue as in Year 2, 3 and 4 but with appropriate numbers.

Use known number facts and place value to subtract

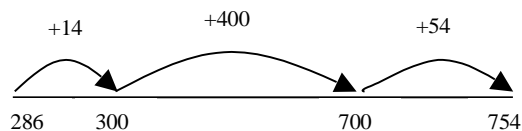
$6.1 - 2.4 = 3.7$



Pencil and paper procedures

Complementary addition

$754 - 286 = 468$



YEAR 6

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $8000 - 2785 = 5215$

To make this method more efficient, the number of steps should be reduced to a minimum through children knowing:

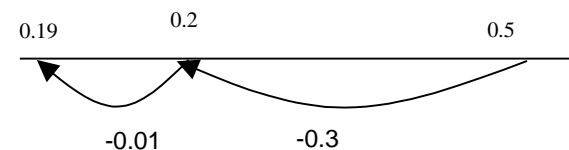
- Complements to 1, involving decimals to two decimal places ($0.16 + 0.84$)
- Complements to 10, 100 and 1000

Subtract the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Year 2, 3, 4 and 5 but with appropriate numbers.

Use known number facts and place value to subtract

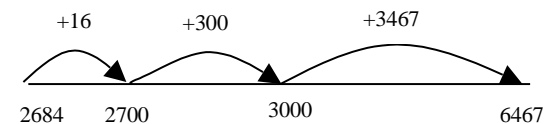
$0.5 - 0.31 = 0.19$



Pencil and paper procedures

Complementary addition

$6467 - 2684 = 3783$



Subtraction		
YEAR 4	YEAR 5	YEAR 6
<p>For those children with a secure mental image of the number line they could record the jumps only:</p> <p>754 – 86 = 668</p> $\begin{array}{r} 14 \text{ (100)} \\ 600 \text{ (700)} \\ \underline{54 \text{ (754)}} \\ 668 \end{array}$ <p>By the end of Year 4 column subtraction using decomposition is developed using 2 and 3 digit numbers e.g. 87- 49</p> $\begin{array}{r} \overset{7}{\cancel{8}}7 \\ - 49 \\ \hline 38 \end{array}$ <p>265 – 148</p> $\begin{array}{r} \overset{5}{2\cancel{6}}5 \\ - 148 \\ \hline 117 \end{array}$	<p>OR</p> <p>754 - 286 = 468</p> $\begin{array}{r} 14 \text{ (300)} \\ 400 \text{ (700)} \\ \underline{54 \text{ (754)}} \\ 468 \end{array}$ <p>14 (300) can be refined to 14 (300) 400 (700) can be refined to 454 (754) 54 (754) can be refined to 468</p> <p>Reduce the number of steps to make the calculation more efficient. <i>Extend to decimals to 1 decimal places</i></p> <p>By the end of Year 5 column subtraction using decomposition is developed using up to 4 digits and decimals e.g. 4325 – 2678</p> $\begin{array}{r} \overset{3}{\cancel{4}}\overset{2}{\cancel{3}}\overset{1}{\cancel{2}}5 \\ - 2678 \\ \hline 1647 \end{array}$ <p>e.g. 52.5 – 26.8</p> $\begin{array}{r} \overset{4}{\cancel{5}}\overset{11}{2}.5 \\ \underline{26.8} \\ 25.7 \end{array}$	<p>OR</p> <p>6467 – 2684 = 3783</p> $\begin{array}{r} 16 \text{ (2700)} \\ 300 \text{ (3000)} \\ \underline{3467 \text{ (6467)}} \\ 3783 \end{array}$ <p>16 (2700) can be refined to 316 (3000) 300 (3000) can be refined to 3467 (6467) 3467 (6467) can be refined to 3783</p> <p>Reduce the number of steps to make the calculation more efficient. <i>Extend to 2 places of decimals</i></p> <p>e.g. 16375 - 5789</p> $\begin{array}{r} \overset{5}{\cancel{16}}\overset{2}{\cancel{3}}\overset{6}{\cancel{7}}5 \\ - 5789 \\ \hline 10586 \end{array}$ <p>e.g. 67.45 – 34.29</p> $\begin{array}{r} 67.\overset{3}{\cancel{4}}5 \\ \underline{34.29} \\ 33.16 \end{array}$ <p>e.g. 4000 - 2474</p> $\begin{array}{r} \overset{3}{\cancel{4}}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}0 \\ \underline{2474} \\ 1526 \end{array}$

Multiplication

YEAR 1

Multiplication is related to doubling and counting groups of the same size.



Looking at columns
 $2 + 2 + 2$
 3 groups of 2

Looking at rows
 $3 + 3$
 2 groups of 3

Counting using a variety of practical resources

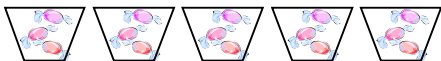
Counting in 2s e.g. counting socks, shoes, animal's legs...

Counting in 5s e.g. counting fingers, fingers in gloves, toes...

Counting in 10s e.g. fingers, toes...

Pictures / marks

There are 3 sweets in one bag.
 How many sweets are there in 5 bags?



YEAR 2

x = signs and missing numbers

$$7 \times 2 = \square \quad \square = 2 \times 7$$

$$7 \times \square = 14 \quad 14 = \square \times 7$$

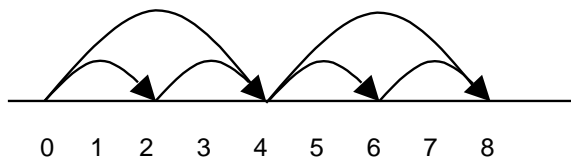
$$\square \times 2 = 14 \quad 14 = 2 \times \square$$

$$\square \times \nabla = 14 \quad 14 = \square \times \nabla$$

Arrays and repeated addition

4×2 or $4 + 4$

2×4 or $2 + 2 + 2 + 2$



Doubling multiples of 5 up to 50

$$15 \times 2 = 30$$

Partition

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: $6 = 5 + 1$ so e.g. Double 6 is the same as double five add double one.



AND double 15

$$10 + 5$$

$$20 + 10 = 30$$

or

X	10	5	
	20	10	= 30

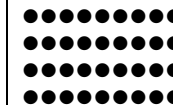
YEAR 3

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).



$$9 \times 4 = 36$$

$$6 \times 3 =$$



Doubling multiples of 5 up to 50

$$35 \times 2 = 70$$

Partition

X	30	5	
	60	10	= 70

Use known facts and place value to carry out simple multiplications

Use the same method as above (partitioning),

e.g. $32 \times 3 = 96$

X	30	2	
	90	6	= 96

Multiplication

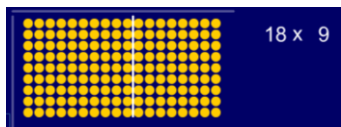
YEAR 4

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Partition

Continue to use arrays:



$$18 \times 9 = 162$$

$$18 \times 9 = (10 \times 9) + (8 \times 9) = 162$$

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

Grid method

23 x 7 is approximately 20 x 10 = 200

x	20	3	=	161
7	140	21		

YEAR 5

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Partition

$$47 \times 6 = 282$$

$$47 \times 6 = (40 \times 6) + (7 \times 6) = 282$$

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

Grid method

72 x 38 is approximately 70 x 40 = 2800

x	70	2	=	2160
30	2100	60		
8	560	16	=	<u>576</u>
				2736

Extend to simple decimals with one decimal place.

e.g. 4.9 x 3

x	4	0.9	=	14.7
3	12	2.7		

This can be extended to formal column multiplication up to 2 digits x 2 digits e.g. 72 x 38

$$\begin{array}{r}
 72 \\
 \times 38 \\
 \hline
 16 \text{ (8x2)} \\
 560 \text{ (8x70)} \\
 60 \text{ (30x2)} \\
 \underline{2100} \text{ (30x70)} \\
 \underline{2736} \\
 \hline
 \end{array}$$

YEAR 6

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Partition

$$87 \times 6 = 522$$

$$87 \times 6 = (80 \times 6) + (7 \times 6) = 522$$

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

Grid method

372 x 24 is approximately 400 x 20 = 8000

x	300	70	2	=	7440
20	6000	1400	40		
4	1200	280	8	=	<u>1488</u>
					<u>8928</u>

Extend to decimals with up to two decimal places.

e.g. 4.92 x 3.2

x	4	0.9	0.02	=	14.76
3	12	2.7	0.06		
0.2	0.8	0.18	0.004	=	<u>0.984</u>
					<u>15.744</u>

Formal multiplication and the compact method are taught in Year 6 e.g. 138 x 72

$$\begin{array}{r}
 138 \\
 \times 72 \\
 \hline
 16 \text{ (2x8)} \\
 60 \text{ (2x30)} \\
 200 \text{ (2x100)} \\
 560 \text{ (70x8)} \\
 2100 \text{ (70x30)} \\
 \underline{7000} \text{ (70x100)} \\
 \hline
 9936
 \end{array}$$

Multiplication - continued

YEAR 4	YEAR 5	YEAR 6
	<p>Developing to the compact method of 2 digits x 1 digit e.g. 45x 6</p> $\begin{array}{r} 45 \\ \times 6 \\ \hline 30 \\ \underline{240} \\ 270 \end{array}$ $\begin{array}{r} 45 \\ \times 6 \\ \hline 270 \\ \substack{3} \end{array}$ <p>e.g. 16.8 x 3</p> $\begin{array}{r} 16.8 \\ \times 3 \\ \hline 2.4 \text{ (3x0.8)} \\ 18 \text{ (3x6)} \\ \underline{30 \text{ (3x10)}} \\ 50.4 \\ \substack{1} \end{array}$	<p>Developing to the compact method of 3digits x 2 digit e.g. 138 x 72</p> $\begin{array}{r} 138 \\ \times 72 \\ \hline 276 \\ \underline{9260} \\ 9936 \\ \substack{1} \end{array}$ <p>e.g. 12.5 x 2.5</p> $\begin{array}{r} 12.5 \\ \times 2.5 \\ \hline 0.25 \text{ (0.5x0.5)} \\ 1 \text{ (0.5x2)} \\ 5 \text{ (0.5x10)} \\ 1 \text{ (2x0.5)} \\ 4 \text{ (2x2)} \\ \underline{20 \text{ (2x10)}} \\ 31.25 \end{array}$ $\begin{array}{r} 12.5 \\ \times 2.5 \\ \hline 6.25 \\ \underline{25} \\ 31.25 \\ \substack{1} \end{array}$

Division

YEAR 1

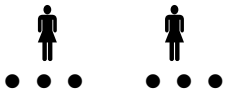
YEAR 2

YEAR 3

Sharing

Requires secure counting skills
-see counting and understanding number strand
Develops importance of one-to-one correspondence
See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

Grouping

Sorting objects into 2s / 3s/ 4s etc
How many pairs of socks are there?



There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?
Jo has 12 Lego wheels. How many cars can she make?

÷ = signs and missing numbers

$$6 \div 2 = \square \quad \square = 6 \div 2$$

$$6 \div \square = 3 \quad 3 = 6 \div \square$$

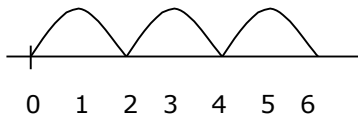
$$\square \div 2 = 3 \quad 3 = \square \div 2$$

$$\square \div \nabla = 3 \quad 3 = \square \div \nabla$$

Grouping

Link to counting and understanding number strand
Count up to 100 objects by grouping them and counting in tens, fives or twos;...
Find one half, one quarter and three quarters of shapes and sets of objects
 $6 \div 2$ can be modelled as:
There are 6 strawberries.
How many people can have 2 each? How many 2s make 6?

$6 \div 2$ can be modelled as:



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



Remainders

In practical problem solving, pupils should recognise that division can result in remainders e.g. when 13 biscuits are shared between 5 children, each child will have 2 biscuits with 3 left over.

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

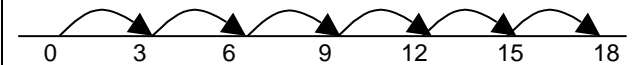
Understand division as sharing and grouping

$18 \div 3$ can be modelled as:

Sharing – 18 shared between 3 (see Year 1 diagram)

OR

Grouping - How many 3's make 18?

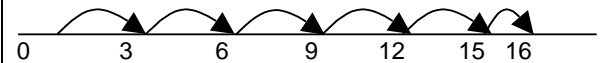


Remainders

$$16 \div 3 = 5 \text{ r}1$$

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over? e.g.



Division

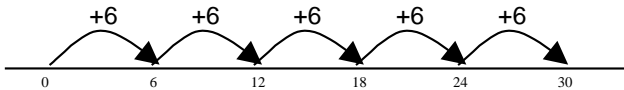
YEAR 4

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Sharing and grouping

$30 \div 6$ can be modelled as:
grouping – groups of 6 placed on no. line and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

Remainders

$41 \div 4 = 10 \text{ r}1$



$41 = (10 \times 4) + 1$

Pencil and paper procedures - chunking

$72 \div 5$ lies between $50 \div 5 = 10$ and $100 \div 5 = 20$

* Partition the dividend into multiples of the divisor:

e.g. $72 = 50 + 22$
 $50 \div 5 = 10$
 $22 \div 5 = 4\text{r}2 \rightarrow 10 + 4\text{r}2 = 14 \text{ r}2$
 OR

$$\begin{array}{r} 72 \\ - \underline{50} \quad (10 \text{ groups}) \\ 22 \\ - \underline{20} \quad (4 \text{ groups}) \\ 2 \end{array}$$

Answer : 14 remainder 2

YEAR 5

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions
 $61 \div 4 = 15 \frac{1}{4}$ or 15.25



Pencil and paper procedures

$256 \div 7$ lies between $210 \div 7 = 30$ and $280 \div 7 = 40$

* Partition the dividend into multiples of the divisor:

e.g. $256 = 210 + 46$
 $210 \div 7 = 30$
 $46 \div 7 = 6\text{r}4 \rightarrow 30 + 6\text{r}4 = 36\text{r}4$

OR

$$\begin{array}{r} 256 \\ - \underline{210} \quad (30 \text{ groups}) \\ 46 \\ - \underline{42} \quad (6 \text{ groups}) \\ 4 \end{array}$$

Answer: 36 remainder 4

YEAR 6

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions
 $676 \div 8 = 84.5$



Pencil and paper procedures

$977 \div 36$ is approximately $1000 \div 40 = 25$

* Partition the dividend into multiples of the divisor:
e.g.

$$\begin{array}{r} 977 \\ - \underline{720} \quad (20 \text{ groups}) \\ 257 \\ - \underline{180} \quad (5 \text{ groups}) \\ 77 \\ - \underline{72} \quad (2 \text{ groups}) \\ 5 \end{array}$$

Answer: $27 \frac{5}{36}$

Division		
YEAR 4	YEAR 5	YEAR 6
<p>This can also be developed to more formal methods for 2 digit numbers divided by 1 digit numbers including remainders e.g. $72 \div 3$</p> $\begin{array}{r} 24 \\ 3 \overline{) 72} \\ \underline{-30} \quad (10 \times 3) \\ 42 \\ \underline{-30} \quad (10 \times 3) \\ 12 \\ \underline{-12} \quad (4 \times 3) \\ 0 \quad 24 \end{array}$ <p>e.g. $96 \div 6$</p> $\begin{array}{r} 16 \text{ r } 1 \\ 6 \overline{) 96} \\ \underline{-60} \quad (10 \times 6) \\ 36 \\ \underline{-36} \quad (6 \times 6) \\ 1 \quad 16 \end{array}$	<p>This can also be developed to more formal methods for 3 digit numbers divided by 1 digit numbers including remainders</p> $\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{-180} \quad (30 \times 6) \\ 16 \\ - \underline{12} \quad (2 \times 6) \\ 4 \quad 32 \end{array}$ <p>Progression leads to the formal compact method</p> <p>e.g. $291 \div 3$</p> $3 \overline{) 291} \begin{array}{l} 0.97 \\ 29 \text{ r } 1 \end{array}$ <p>e.g. $436 \div 7$</p> $\begin{array}{r} 0 \text{ 6 } 2 \text{ r } 2 \\ 7 \overline{) 436} \\ \underline{43} \text{ 6} \\ 16 \end{array}$	<p>Continue to develop division by 2 digit numbers e.g. $684 \div 33$</p> $\begin{array}{r} 20 \text{ r } 24 \\ 33 \overline{) 684} \\ \underline{-330} \quad (10 \times 33) \\ 354 \\ \underline{-330} \quad (10 \times 33) \\ 024 \quad 20 \end{array}$ <p>Or</p> $0 \text{ 2 } 0 \text{ r } 24$ $33 \overline{) 68 \text{ r } 4}$ <p>Continue to develop the compact method including remainders expressed as decimals and fractions</p> <p>e.g. $1473 \div 4$</p> $\begin{array}{r} 0 \text{ 3 } 6 \text{ 8 } . 25 \\ 4 \overline{) 1473} \\ \underline{14} \text{ 7 } 3 \\ \underline{14} \text{ 7 } 3 \\ 0 \text{ 2 } 0 \end{array}$ <p>Or</p> $\begin{array}{r} 0 \text{ 3 } 6 \text{ 8 } \text{ r } \frac{1}{4} \\ 4 \overline{) 1473} \\ \underline{14} \text{ 7 } 3 \end{array}$